The 3rd Northeast Mathematics Undergraduate Research Mini-Symposium

July 28, 2015

University of Connecticut

Scientific Program & Book of Abstracts

Organizers

Luke G. Rogers & Joe P. Chen

The 3rd Northeast Mathematics Undergraduate Research Mini-Symposium
University of Connecticut, July 28, 2015

Full schedule

Time	Agenda & Location
9:30∼10:40	**Registration & Poster Session** *Laurel Hall Foyer*
10:40∼11:40	**Student Talks: Morning Session I** *Laurel Hall Rooms 301, 302, 305*
11:40∼11:50	Break
11:50∼12:50	**Student Talks: Morning Session II** *Laurel Hall Rooms 301, 302, 305*
12:50∼1:00	**Photo-op**
1:00∼2:00	**Lunch** *Student Union Room 304*
2:10∼2:40	**Panel on applying to graduate schools** *Laurel Hall Room 101*
2:40∼3:40	**Poster Session** *Laurel Hall Foyer*
3:40∼4:20	**Student Talks: Afternoon Session** *Laurel Hall Rooms 301, 302, 305*
4:30∼	Departure

Participating schools

Amherst · Columbia · Fairfield · Mount Holyoke · Smith
UConn · UConn Health · UMass · Williams · Yale

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University of Connecticut, July 28, 2015

Schedule of Talks

Morning Session I (10:40∼11:40)

Laurel Hall Room 301 (Session Chair: Joe Chen)	
Time	Title & Presenters
10:40∼11:00	**Limiting distributions for topological Markov chains with holes** Ianzano, Meyer, Morfe, Yoo *(Fairfield)*
11:00∼11:20	**PageRank algorithm for multi-way graph partitioning** Carpenter-Winch, Sayde *(Columbia)*
11:20∼11:40	**Weighted Graph Partitoning Using PageRank Vectors** Dung, Potter, Shah *(Columbia)*

Laurel Hall Room 302 (Session Chair: Khrystyna Serhiyenko)	
Time	Title & Presenters
10:40∼11:00	**Counting subrings of $\mathbb{Z}^N$** Atanasov, Krakoff, Menzel *(Yale)*
11:00∼11:20	**Minimal Length Maximal Green Sequences for Type $\mathbb{A}$ Quivers** Cormier, Dillery, Resh, Whelan *(UConn)*
11:20∼11:40	**Combinatorial quantum modular forms** Ki, Truong Vu, Yang *(Amherst)*

Laurel Hall Room 305 (Session Chair: Tom Laetsch)	
Time	Title & Presenters
10:40∼11:00	**The combinatorics of Borel-fixed sets in a fixed degree** Dennis, Santiago *(Mt. Holyoke)*
11:00∼11:20	**Obstructions to Symplectic Embeddings** Huq-Kuruvilla, Sultani *(Columbia)*
11:20∼11:40	**Volume estimates for certain hyperbolic 3-dimensional orbifolds** Mallepalle, Melby, Vaccaro *(Fairfield)*

Schedule of Talks

Morning Session II (11:50∼12:50)

Laurel Hall Room 301 (Session Chair: Fan Ny Shum)	
Time	Title & Presenters
11:50∼12:10	**Stochastic Stabilization of Multivariable Systems in $\mathbb{C}$** Majumdar, McCain, O'Connell *(UConn)*
12:10∼12:30	**On the Numerators of Hurwitz Numbers** Hamann, Varadaraj *(Columbia)*
12:30∼12:50	**Relationship of the Hennings and Chern-Simons Invariants For Higher Rank Quantum Groups** Cheong, Doser, Gray *(Fairfield)*

Laurel Hall Room 302 (Session Chair: Antoni Brzoska)	
Time	Title & Presenters
11:50∼12:10	**A Ramsey Theoretic Approach to Function Fields and Quaternions** Asada, Manski *(Williams)*
12:10∼12:30	**Pure Simplicial Complexes and Cones of Divisors** Gunther, Zhang *(Yale)*
12:30∼12:50	**Magnetic Spectral Decimation on the Diamond Fractal** Hansalik, Loew *(UConn)*

Laurel Hall Room 305 (Session Chair: Ulysses Andrews)	
Time	Title & Presenters
11:50∼12:10	**Counting 10-arcs in the Projective Plane** Lawrance, Peilen, Weinreich *(Yale)*
12:10∼12:30	**Fractal Alternating Current Circuits** Anderson, Davis *(UConn)*
12:30∼12:50	**Query Complexity of Mastermind Variants** Berger, Chute, Stone *(Yale)*

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Schedule of Talks

Afternoon Session (3:40∼4:20)

Laurel Hall Room 301 (Session Chair: Susie Kimport)	
Time	Title & Presenters
3:40∼4:00	**Extensions to Bidding Games** Gaitonde, Lee, Pasternak *(Yale)*
4:00∼4:20	**Zero-Bids in a Two Player Product-Mix Auction** Brandfonbrener, Levmore, Zanger-Tishler *(Yale)*

Laurel Hall Room 302 (Session Chair: Kyle Evans)	
Time	Title & Presenters
3:40∼4:00	**Investigating the Interplay of Argumentation and Mathematics in Classroom Tasks** Brown, Wright *(UConn)*
4:00∼4:20	**Mathematics and Intercultural Competence in the Middle School** Bennett, Brunner *(UConn)*

Laurel Hall Room 305 (Session Chair: Jose Gonzalez)	
Time	Title & Presenters
3:40∼4:00	**Geometry of Lattices Under Incompressible Flows: An Application to Molecular Dynamics** Geraldo *(UMass)*
4:00∼4:20	**Generalized Splines over $\mathbb{Z}$ and $\mathbb{Z}/m\mathbb{Z}$** Philbin *(Smith)*

List of oral presentations

Combinatorial quantum modular forms

(Amherst College)
Caleb Ki, *Amherst College*
Yen Nhi Truong Vu, *Amherst College*
Bowen Yang, *Amherst College*

Abstract

Quantum modular forms, defined by Zagier in 2010, have been studied by many authors including Bringmann, Folsom, Ono, Rhoades, Rolen and others within the last few years. In this talk, we will define new quantum modular forms which are intimately related to strongly unimodal sequences of integers, and will discuss their combinatorial and analytic properties.

Obstructions to Symplectic Embeddings

(Columbia University)
Irit Huq-Kuruvilla, *Columbia University*
Nawaz Sultani, *Columbia University*

Abstract

Ever since Gromov proved his famous nonsqueezing theorem, the question of when can a symplectic manifold be symplectically embedded into another has been widely studied. In this talk, we introduce the basics of sympelctic geometry, the nonsqueezing theorem, and some combinatorial techniques for obstructing the embeddings for sympelctic toric domains. We talk about a theorem of Hutchings regarding these sorts of problems, and the results we obtained from applying this theorem to certain embedding problems.

On the Numerators of Hurwitz Numbers

(Columbia University)
Linus Hamann, *Columbia University*
Srikar Varadaraj, *Columbia University*

Abstract

Hurwitz Numbers are a sequence of rational numbers analogous to Bernoulli Numbers for the field $\mathbb{Q}(i)$. A result of Katz completely describes the denominators of these numbers. In this paper, we attempt to carry out the same analysis for the behavior of the numerators. It is revealed that the behavior of a prime in the numerator depends on whether p is congruent to 1 or 3 mod 4, corresponding to the splitting of primes in $\mathbb{Q}(i)$. We show that in the case of p congruent to 1 mod 4 the behavior is very similar to that of the sporadic behavior of the primes in the numerator of Bernoulli Numbers. In the case of p congruent to 3 mod 4, a much nicer behavior is revealed and an explicit formula is proven.

Organizers' note: Also see the poster presentation by Kevin Choi (Columbia) and Xin Xu (Columbia).

PageRank algorithm for multi-way graph partitioning

(Columbia University)

Louisa Carpenter-Winch, *Columbia University*
Roxane Sayde, *Barnard College*

Abstract

In order to detect underlying community structure in graphs, one can partition them into highly connected subgraphs. Google's PageRank algorithm is a local method of making a cut that is both sparse and balances the volume of two subgraphs. We analyze an iterated version of the algorithm for multi-way graph partitioning, as it performs on graphs generated with the stochastic block model.

Weighted Graph Partitoning Using PageRank Vectors

(Columbia University)

Nguyen Chi Dung, *Columbia University*
Greyson Potter, *Columbia University*
Hardik Shah, *Columbia University*

Abstract

The problem of local graph partitioning for unweighted graphs through the use of PageRank vectors has been well explored. In this presentation, we'll state some parallel results for the weighted graph case, and talk about some of the difficulties that occur when trying to implement the Approximate PageRank vector algorithm for weighted graphs. We provide guarantees on the conductance and volume of the support of the partition equivalent to those of the unweighted case, and some relatively sharp bounds on the runtime of the algorithm.

Limiting distributions for topological Markov chains with holes

(Fairfield University)

Christopher Ianzano, *SUNY Stonybrook*
Philip Mayer, *Fairfield University*
Peter Morfe, *Cooper Union*
Elizabeth Yoo, *Columbia University*

Abstract

Open dynamical systems are models of physical systems in which mass or energy is allowed to escape from the system. Central questions involve the existence of conditional equilibria (measures that are invariant under the dynamics conditioned on non-escape) which can be realized as limiting distributions under the dynamics of the open system. We study this problem in the context of topological Markov chains, which are a class of symbolic dynamical systems with a wide variety of applications. Under a combinatorial condition on the Markov chain and for positive recurrent potentials, we prove the existence (and uniqueness in a certain class) of limiting distributions which represent conditional equilibria for the open system. We also prove a relation between the escape rate from the system and the entropy on the survivor set (the set of points that never enters the hole).

Relationship of the Hennings and Chern-Simons Invariants For Higher Rank Quantum Groups

(Fairfield University)

Winston Cheong, *Rowan University*
Alex Doser, *Iowa State University*
McKinley Gray, *SUNY Geneseo*

Abstract

The Hennings and Chern-Simons invariants are two different invariants of three-manifolds, *i.e.* tools for distinguishing three-dimensional spaces, which can be constructed by similar means from the same starting point, an algebraic object called a quantum group. The latter is reasonably well understood and deeply connected to important physics, but the former is quite mysterious. Recently Chen, Yu and Zhang were able to show that these two invariants had a simple relationship, in fact that the second determines the first, when the invariants are constructed from the simplest quantum group. We partially generalized this result for arbitrary quantum group, showing that the invariants agree on an important subclass of three-manifolds, integral homology three-spheres.

Volume estimates for certain hyperbolic 3-dimensional orbifolds

(Fairfield University)

Jessica Mallepalle, *Arcadia University*
Joseph Melby, *University of Minnesota, Morris*
Jennifer Vaccaro, *Olin College of Engineering*

Abstract

A hyperbolic 3-orbifold is a 3-dimensional space with a geometric structure that obeys the laws of hyperbolic geometry. A key invariant of a hyperbolic 3-orbifold is its hyperbolic volume. For this project, we use estimation techniques due to Agol, Storm, and Thurston to provide lower bounds on the volume of a class of hyperbolic 3-orbifolds, based on whether they contain certain types of 2-dimensional hyperbolic suborbifold.

The combinatorics of Borel-fixed sets in a fixed degree

(Mount Holyoke College)

Rose Dennis, *Mount Holyoke College*
Cecily Santiago, *Mount Holyoke College*

Abstract

We use polynomials to model many phenomena in the natural world. Monomials are polynomials with one term, and their study can reveal properties of polynomial functions. We focus on sets of monomials that satisfy the Borel property, called Borel-fixed sets.

If for any monomial m in n variables $x_1 > x_2 > ... > x_n$ in a set S, whenever $x_j | m$ and $x_i > x_j$, $m\frac{x_i}{x_j}$ is also in S, then we say that S satisfies the Borel property. We refer to S as a Borel-fixed set. Expanding our knowledge of Borel-fixed sets may translate to a better understanding of polynomial functions.

Our research investigates the structure of Borel-fixed sets in terms of partial ordering. We demonstrate that the partially ordered set of all monomials of degree d in n variables is a lattice, and is graded with rank function $r(m = x_1^{a_1} x_2^{a_2} ... x_n^{a_n}) = \sum_{i=1}^{n} (i-1)a_i$. Additionally, we consider methods for counting the number of Borel-fixed sets of degree d in n variables, and show that there are $d+1$ Borel-fixed sets of degree d in two variables and $2^{d+1} - 1$ Borel-fixed sets of degree d in three variables.

Generalized Splines over $\mathbb{Z}$ and $\mathbb{Z}/m\mathbb{Z}$

(Smith College)

McCleary Philbin, *Smith College*

Abstract

Fix a ring, R, and label the edges of a connected graph, G, with ideals in R. A generalized spline on G over R is a set of vertex labels in R that satisfy the condition that whenever two vertices are joined by an edge, the labels on those vertices differ by a multiple of the label on the edge by which they are joined. The collection of all splines on G forms a module, the specifics of which vary significantly depending on our choice of R. In this talk, we give a survey of recent work on generalized splines in two closely related cases: when our ring is $\mathbb{Z}$ and when it is $\mathbb{Z}/m\mathbb{Z}$.

Fractal Alternating Current Circuits

(University of Connecticut)

Loren Anderson, *North Dakota State University*

Hannah Davis, *University of Minnesota*

Abstract

The infinite ladder inductor-capacitor (LC) circuit, explored by Feynman, was historically used as a prototypical model for wave propagation in one dimension. We construct and analyze the properties of analogous ladder circuits based on self-similar graphs. Fractal circuits involving pure resistors have been well studied; therefore, we examine the more complicated case involving inductors and capacitors, which have complex impedances.

To this end, we consider modified Sierpinski Gasket ladders and Hanoi Tower-like circuits. In each case, we compute the effective impedance of the infinite circuit using Kirchhoff's circuit laws. From this, we are able to determine the filter condition - the parameters for which the effective impedance has a positive real part - which implies that a traveling wave on the circuit will attenuate. We also derive a harmonic extension algorithm which enables us to solve the potential problem on any level of the circuit. Finally, we find the energy forms corresponding to real and complex power dissipation on these circuits and verify that they obey the laws of energy conservation. Our discussion of energy forms on these fractal circuits serves as a springboard for analyzing the more difficult problem of wave propagation on fractals.

Investigating the Interplay of Argumentation and Mathematics in Classroom Tasks

(University of Connecticut)

Megan Brown, *University of Dayton*

Grace Wright, *Bates College*

Abstract

In this presentation, we will discuss the results of a study motivated by the current emphasis on argumentation in the new Common Core State Standards of Mathematics. In particular, we focused on tasks that address the third Standard for Mathematical Practice that highlights argumentation as an expertise all students should develop. Our data consists of 157 tasks from 40 elementary and high school teachers participating in a professional development program focused on mathematical argumentation. We analyzed how argumentation affects the cognitive demand levels of the tasks, as well as how argumentation detracts or contributes to the mathematical content in the tasks. Our findings show a change in cognitive demand levels when tasks were analyzed with and without the argumentation component. In addition, five different themes emerged from our analysis with regards to the interplay of argumentation and mathematical concepts, some of which were common across elementary and high school tasks.

Magnetic Spectral Decimation on the Diamond Fractal

(University of Connecticut)

Madeline Hansalik, *Texas A&M University*
Stephen Loew, *Coe College*

Abstract

The Laplacians for a large class of self-similar fractals and fractal graphs exhibit a property called spectral decimation, in which the spectra of different levels of approximation are related by a dynamical system involving a rational function. Expanding upon the work of Malozemov and Teplyaev [1], we extend some aspects of the spectral decimation method from the Laplacian operator to a magnetic Laplacian operator, and use this to numerically investigate properties of the magnetic spectrum of this operator on the diamond fractal. In particular, we identify the correct unitary transformations and projections to obtain the aforementioned rational functions.

[1] Leonid Malozemov, Leonid; Teplyaev, Alexander "Self-Similarity Operators, Operators, and Dynamics." *Mathematical Physics, Analysis and Geometry* Volume 6, Issue 3 (2003).

Organizers' note: Also see the poster presentation by Aubrey Coffey (Agnes Scott).

Mathematics and Intercultural Competence in the Middle School

(University of Connecticut)

Christopher Bennett, *Sacred Heart University*
Megan Brunner, *SUNY Geneseo*

Abstract

As today's world becomes increasingly globalized, there exists a greater need to develop intercultural competence (ICC) in children through education. Through a partnership with the Farmington (CT) School District, three interdisciplinary units were designed to develop ICC in sixth grade students, but lack assessment tools that track this development over the course of a school year. With a focus on Michael Byram's model of ICC, we created two assessment tools - a survey to assess attitudes towards and knowledge of cultures and learning across content areas, and a rubric to assess interactions and reflections - that we plan to use in Farmington in the upcoming year. We are also interested in the relationship between math and ICC, and created lesson plans for 3rd, 6th, and 9th grades that serve as an example and incorporate ICC and the Common Core standards.

Minimal Length Maximal Green Sequences for Type $\mathbb{A}$ Quivers

(University of Connecticut)

Emily Cormier, *Bowdoin College*
Peter Dillery, *University of Virginia*
Jill Resh, *Roger Williams University*
John Whelan, *Vassar College*

Abstract

The study of maximal green sequences (MGS) is motivated by string theory, in particular Donaldson-Thomas invariants and the BPS spectrum. This concept can also be examined through the framework of τ-tilting modules in representation theory. It is known that triangulations of disks with no punctures yield type $\mathbb{A}$ quivers. B. Keller introduced green mutations and the corresponding MGS's. These sequences can be studied both through the combinatorial transformations of directed graphs as well as through triangulations of disks.

Our research focuses on maximal green sequences of minimal length for quivers mutation equivalent to type $\mathbb{A}$ quivers. It is known that each acyclic quiver has at least one minimal length MGS of length n, where n is the number of vertices in the quiver. First, we define an algorithm that produces such a sequence of mutations for any given acyclic type $\mathbb{A}$ quiver. For cyclic type $\mathbb{A}$ quivers, we define an algorithm that produces an MGS of length $n+t$ where n is the number of vertices and t is the number of 3-cycles in a quiver. We then proceed to show that $n+t$ is always the minimal length of MGS's corresponding to any type $\mathbb{A}$ quiver.

Stochastic Stabilization of Multivariable Systems in $\mathbb{C}$

(University of Connecticut)

Rajeshwari Majumdar, *University of Connecticut*
Heather McCain, *Schreiner University*
Dylan O'Connell, *Haverford College*

Abstract

When a single-variable complex-valued polynomial Ordinary Differential Equation (ODE) has trajectories blowing up in finite time (which we will refer to as *explosions*), the addition of a suitable complex-valued Brownian motion can stabilize the trajectories from any initial condition in $\mathbb{C}$ toward the stable fixed point with probability 1. However, much less is understood about the analogous stochastic stabilization problem in higher dimensions. To that end, we investigate a prototype multivariable system of ODEs in $\mathbb{C}^2$ of the form

$$\begin{cases} \dot{z}_t = -\nu z_t + \alpha z_t w_t \\ \dot{w}_t = -\nu w_t + \beta z_t w_t \quad \text{where } \alpha, \beta \in \mathbb{R},\ \nu \in \mathbb{R}^+ \\ z_0 \in \mathbb{C},\ w_0 \in \mathbb{C} \end{cases}$$

which admits explosive solutions. We found that by performing a coordinate transformation, our system can be reduced to a quasi-1-dimensional ODE; this modification enables us to identify necessary and sufficient conditions for an additive Brownian noise to stabilize the system. These conditions have been verified numerically, and rigorous proofs are forthcoming.

Geometry of Lattices Under Incompressible Flows: An Application to Molecular Dynamics

(University of Massachusetts, Amherst)

Abdel Kader Geraldo, *University of Massachusetts, Amherst*

Abstract

We consider the problem of lattices deforming under incompressible flow that arises when simulating fluid flow at a molecular level. The lattices considered define the simulation domain and its periodicity. For certain choices lattice orientation, the simulation can break down after a finite amount of time as a particle and its periodic replicas approach arbitrarily close. We will talk about how this is avoided in general cases and discuss an extension to degenerate flows.

A Ramsey Theoretic Approach to Function Fields and Quaternions

(Williams College)

Megumi Asada, *Williams College*
Sarah Manski, *Kalamazoo College*

Abstract

Ramsey theory concerns itself with how large a set needs to be for a certain structure to arise. The classic Friends and Strangers problem asks how many people need to be at a party to guarantee 3 mutual friends or 3 mutual strangers. Ramsey theory can also be applied to understanding the structure of algebraic rings. Rankin's 1961 paper formed a bridge between Ramsey Theory and Number Theory by greedily constructing a set of integers avoiding 3-term geometric progressions. This was generalized last year to integer rings of number fields. In our work, we resolve analogous problems for polynomials over finite fields and quaternions in the Hurwitz order.

We construct, for any $\mathbb{F}_q[x]$, a large greedy set of polynomials free of 3-term progressions. We also give upper and lower bounds on the supremum of upper densities of 3-term progression-free sets. Next, we apply this problem to the Hurwitz Quaternions, where the loss of commutativity greatly complicates the arguments and affects the limiting behavior. Though most famous for their applications in physics and geometry, the Hurwitz order of quaternions possesses other interesting algebraic properties. The non-commutative ring contains 24 units and an interesting Metacommutation Problem that allows prime factorizations of quaternions that aren't unique in the classical sense of the integers. We construct maximally sized sets of Hurwitz quaternions that avoid geometric progressions up to units.

Joint with Eva Fourakis, Eli Goldstein, Nathan McNew, Steven J. Miller, and Gwyn Moreland.

Counting 10-arcs in the Projective Plane

(Yale University)

Rachel Lawrence, *Yale University*
Luke Peilen, *Yale University*
Max Weinreich, *Yale University*

Abstract

An n-arc in the projective plane over a finite field is a collection of n distinct points in the plane, no three of which lie on a line. Formulae for the number of n-arcs in $\mathbb{P}^2(\mathbb{F}_q)$ are known only through $n = 9$. Building on the work of D. Glynn, we exhibit the 151 unique projective $[10_3, n_3]$-superfigurations, and find conditions on q which determine the number of occurrences of each in $\mathbb{P}^2(\mathbb{F}_q)$. Using these results, we determine an expression for the number of 10-arcs in $\mathbb{P}^2(\mathbb{F}_q)$.

Counting subrings of $\mathbb{Z}^N$

(Yale University)

Stanislav Atanasov, *Yale University*
Benjamin Krakoff, *Yale University*
Julian Menzel, *Yale University*

Abstract

We study subrings of finite index of $\mathbb{Z}^N$, where the addition and multiplication and componentwise. If we denote by $f_n(k)$ the number of subrings of index k, we find a formula for all integers k that are not divisible by a 7-th power of a prime, thus extending a result due to Liu. We also show serious advances towards the computation of the generating function $A_5(p,x) = \sum_{e=0} f_5(p^e)x^e$, which is the first outstanding case. We also investigate the number of subrings of $\mathbb{Z}[x]/x^n$, and compute the number of subrings of index k for all integers k not divisible by a 4-th power of a prime.

Extensions to Bidding Games

(Yale University)

Jason Gaitonde, *Yale University*
Seung Hyun Lee, *Yale University*
Charlie Pasternak, *Yale University*

Abstract

Traditional bidding games are combinatorial games in which two players, rather than alternating moves, instead bid in a first-price auction for the right to choose who moves next. We explore two distinct variants of these games. First, to capture the effect of complementary moves, we analyze bidding games in which the auction for the next move is replaced by a combinatorial auction for the next k moves. We find optimal strategies for these games and prove that, as in traditional bidding games, these games have a winning strategy for almost all distributions of money. Second, we examine bidding games with more than two players. We prove structural results about distributions where players have winning strategies, particularly when the game has generic winning strategies.

Pure Simplicial Complexes and Cones of Divisors

(Yale University)

Elijah Gunther, *Yale University*
Olivia Zhang, *Yale University*

Abstract

We study the cone of pseudoeffective divisors of $\bar{M_{0,n}}$, a blow up of projective space. Following the work of Doran, Giansiracusa, and Jensen, we investigate pure-dimensional simiplicial complexes in order to better understand the minimal generators of this cone. Our work primarily relies on combinatorial, topological, and linear algebraic methods along with computer work.

Query Complexity of Mastermind Variants

(Yale University)

Aaron Berger, *Yale University*
Christopher Chute, *Yale University*
Matthew Stone, *Yale University*

Abstract

We consider generalizations of the popular board game "Mastermind." The game consists of two players: the codemaker and the codebreaker. At the start the codemaker chooses a vector $(x_1, x_2, \ldots, x_n)$ with $x_i \in [1, k]$, which is unknown to the codebreaker, and in each turn the codebreaker makes inquiries with the end goal to discover said vector. We will discuss asymptotics for the minimum number of of guesses needed by the codebreaker to guarantee a success. We also analyze some variants of the original game, including not allowing repeated colors and non-adaptive strategies.

Zero-Bids in a Two Player Product-Mix Auction

(Yale University)

David Brandfonbrener, *Yale University*
Eliot Levmore, *Yale University*
Michael Zanger-Tishler, *Yale University*

Abstract

Engelbrecht-Wiggans and Kahn (1998) developed conditions under which bidders in a multi-unit, single good, nth price auction had an equilibrium strategy of pooling bids at zero instead of truthfully bidding their valuations. Klemperer (2008) created a new auction mechanism called the product-mix auction that allows the auctioneer to sell a fixed number of units allocated after the auction among multiple different varieties of goods. Klemperer assumes that the auction will have a large number of bidders and thus that the bidders will bid their valuations. We examine the product-mix auction with small numbers of bidders and find that, like the multi-unit, single good, nth price auction, there are conditions under which bidders will pool their bids at zero rather than truthfully bid their valuations.

List of poster presentations

On the Numerators of Hurwitz Numbers

(Columbia University)
Kevin Choi, *Columbia University*
Xin Xu, *Columbia University*

Abstract

Hurwitz Numbers are a sequence of rational numbers analogous to Bernoulli Numbers for the field $\mathbb{Q}(i)$. A result of Katz completely describes the denominators of these numbers. In this paper, we attempt to carry out the same analysis for the behavior of the numerators. It is revealed that the behavior of a prime in the numerator depends on whether p is congruent to 1 or 3 mod 4, corresponding to the splitting of primes in $\mathbb{Q}(i)$. We show that in the case of p congruent to 1 mod 4 the behavior is very similar to that of the sporadic behavior of the primes in the numerator of Bernoulli Numbers. In the case of p congruent to 3 mod 4, a much nicer behavior is revealed and an explicit formula is proven.

Organizers' note: Also see the oral presentation by Irit Huq-Kuruvilla (Columbia) and Nawaz Sultani (Columbia).

Tuberculosis Disease Modeling in the USA

(Mount Holyoke College)
Olivia Justiynski, *Mount Holyoke College*
Chenyue Lu, *Mount Holyoke College*
Ellie Mainou, *Smith College*

Abstract

Tuberculosis (TB) is a disease of great global epidemiological importance. According to WHO, one third of the world's population has latent TB. Not only is TB prevalence high, but the future of TB management will also have to face the additional challenge of drug resistance. Strains resistant to the most common first-line drugs for TB treatment are already widespread. We constructed a compartmental mathematical model of TB in the US that encompasses four strains of differing resistance to treatment. This model also takes into account the effect of immigration, which contributes to the prevalence of latent TB in the US. We are currently fitting parameters to recent CDC data on TB morbidity and mortality. Our goal is to use this model to predict the future impact of drug-resistant TB on the US and the most effective means of TB control.

Magnetic Spectral Decimation on the Diamond Fractal

(University of Connecticut)

Aubrey Coffey, *Agnes Scott College*

Abstract

The Laplacians for a large class of self-similar fractals and fractal graphs exhibit a property called spectral decimation, in which the spectra of different levels of approximation are related by a dynamical system involving a rational function. Expanding upon the work of Malozemov and Teplyaev [1], we extend some aspects of the spectral decimation method from the Laplacian operator to a magnetic Laplacian operator, and use this to numerically investigate properties of the magnetic spectrum of this operator on the diamond fractal. In particular, we identify the correct unitary transformations and projections to obtain the aforementioned rational functions.

[1] Leonid Malozemov, Leonid; Teplyaev, Alexander "Self-Similarity Operators, Operators, and Dynamics." *Mathematical Physics, Analysis and Geometry* Volume 6, Issue 3 (2003).

Organizers' note: Also see the oral presentation by Madeline Hansalik (Texas A&M) and Stephen Loew (Coe).

Stabilization by Noise in $\mathbb{C}^2$–Valued Nonlinear Systems

(University of Connecticut)

Lance Ford, *University of Central Oklahoma*
Derek Kielty, *Georgia Institute of Technology*

Abstract

The following $\mathbb{C}^2$–valued system has solutions which blow up in finite time:

$$\begin{cases} \frac{dz}{dt} = -\nu z + \alpha z w \\ \frac{dw}{dt} = -\nu w + \beta z w \\ z(0) = z_0 \in \mathbb{C} \\ w(0) = w_0 \in \mathbb{C}, \end{cases} \qquad \text{where } \nu \in \mathbb{R}^+, \alpha, \beta \in \mathbb{R}.$$

By introducing additive Brownian noise to these equations, we can stabilize the entire system. The noise stabilizes the system by pushing unstable solution curves onto stable flow lines. Recent results have shown how additive noise can stabilize complex single–variable systems. In particular, the system is shown to be ergodic and its solutions converge exponentially fast toward a unique invariant measure. We have numerical results that suggest the same results can be produced for our multivariable system. We numerically verified that the addition of Brownian noise stabilized these formerly unstable solutions. Furthermore, we simulated the invariant measure, which has a heavy-tailed distribution.

Wave Propagation through a Fractal Medium

(University of Connecticut)

Edith Aromando, *University of New Hampshire*
Lee Fisher, *Appalachian State University*

Abstract

We consider the wave equation on the unit interval with fractal measure, and use two numerical models to study wave speed and propagation distance. The first approach uses a Fourier series of eigenfunctions of the fractal Laplacian, while the second uses a Markov chain to model the transmission and reflection of classical waves on an approximation of the fractal. These models have complementary advantages and limitations, and we conjecture that they approximate the same fractal wave.

A Compartmental Model of Positive and Negative Stimulation in T Cell Receptor Signaling

(UConn Health, Center for Quantitative Medicine)

Hannah Rollins, *Clemson University*
Alex Galarce, *New College of Florida*
Madison Brandon, *UConn Health, Center for Quantitative Medicine*
Reinhard Laubenbacher, *UConn Health, Center for Quantitative Medicine*

Abstract

Unlike traditional cancer treatment, the goal of Cancer Immunotherapy is to help the immune system, specifically the cancer fighting CD8+ T-cell, fight cancer. The idea was first presented in 1893 and gained traction in the 1970s. An initial activation signal from the T-cell Receptor and a secondary, amplifying signal from costimulatory receptors begin a series of signaling processes that lead to the activation of transcription factors important for the proliferation of CD8+ T-cells. Inhibitory receptors are responsible for terminating this signal and deactivating the T-cell. We present a model of the major intracellular signaling of the T-cell after its activation. Our approach utilizes mathematical modeling and simulation tools such as BioNetGen and Copasi and allows us to simulate different initial conditions of T-cell interactions as well as to further understand the intracellular T-cell activation and deactivation signals. Furthermore, we will explore how the T-cell responds to Cancer Immunotherapy treatments which block the inhibitory receptors.

Pipeline to infer brain connectivity networks from fMRI data

(UConn Health, Center for Quantitative Medicine)

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Abstract

The association between functional connectivity networks and neurological diseases has been established in past studies. Accurately identifying such connections from neuroimaging data, though, is a non-trivial matter, requiring the careful selection of network inference methods, a choice that can be better informed through benchmarking. A systematic comparison of neuroscience network inference methods along with molecular biology and crowd sourced methods was performed by generating and analyzing in silico fMRI time series based on random and scale-free networks. Results were then evaluated with different statistical metrics. Methods from molecular biology performed well, with GENIE consistently scoring in the top 3 methods. Creating consensus networks also proved effective, with a consensus of the top five methods consistently outperforming individual methods. A pipeline for inferring brain connectivity networks should include a consensus network of the top 5 performing methods for improved results.

Whole Body Model of Iron Dynamics

(UConn Health, Center for Quantitative Medicine)

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Abstract

Iron plays an important role in many processes of the body, most importantly oxygen transport by red blood cells. The goal of this research is to create a predictive model of whole body iron metabolism for humans. As a first step, a whole body model of iron homeostasis was created for mice. This model will be used to gain a better understanding of iron metabolism disorders (*i.e.* anemia and hemochromatosis). The present mouse model was calibrated to data previously published by the Reich group. All calculations, including parameter estimation, were carried out with the open-source software COPASI.

Cohen-Lenstra Heuristics for Hyperelliptic Graphs

(University of Massachusetts, Amherst)

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Daniel Normand, *University of Massachusetts, Amherst*
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Abstract

Probabilities for the distribution of p-parts for the Jacobians of graphs are generally known. We examined the distribution of p-parts of the Jacobians of hyperelliptic graphs, a discrete geometric analog of Hyperelliptic Riemann surfaces which are, in turn, a geometric analog of quadratic number fields, in order to determine if the C.L. heuristics are analogous to those for arbitrary graphs. We used databases of simple, connected graphs, developed algorithms to detect which graphs were hyperelliptic and simultaneously computed the Jacobian, then analyzed the distribution of p-parts of the Jacobians of strictly the hyperelliptic graphs. From our results, it appears the Cohen-Lenstra Heuristics do not hold for Hyperelliptic graphs; we are in the process of finding an explanation.