Feynman introduces an infinite ladder LC-circuit in his lecture notes [1]. He finds the impedance of the infinite case by assuming the existence of the limit of the impedance as rungs are added to the ladder.

\[
\begin{align*}
\text{Motivation} & \quad \text{Characteristic Impedance and Filter Conditions} \quad \text{Power Dissipation} \\
\text{Under certain conditions, this impedance has a positive real part, despite the fact that the circuit is built from components with purely imaginary impedances. We consider whether this physically interesting situation has analogues for fractal ladder circuits, specifically the self-similar Sierpinski Gasket ladder circuit (top) and the weakly self-similar Hanoi Tower-like circuit (bottom).} \end{align*}
\]

The characteristic impedance \( z \) of the SG ladder is the impedance between vertices \( p_0 \) and \( p_1 \). It can be computed by replacing infinite portions of the ladder with the characteristic impedance and reducing using Kirchhoff’s laws. The circuit is a filter when it has energy dissipation, i.e. when \( z \) has positive real part.

\[
\text{Theorem 1} \quad \text{If} \quad z_L = \frac{\omega L}{2} \text{and} \quad z_C = \frac{1}{\omega C}, \quad \text{where} \quad \omega = \text{the AC frequency}, \quad \text{then} \quad \text{the SG ladder is a filter when} \quad 9(4 - \sqrt{5}) < 2\omega^2 LC < 9(4 + \sqrt{5}), \quad \text{and} \quad z = \frac{1}{10\omega C} \left(2\omega^2 LC + 9\omega + 14\omega^2 LC - 4(\omega^2 LC)^2 - 8L \right).
\]

In general, the impedance of an infinite LC circuit is not the limit of the impedances of finite approximations to the infinite circuit. However we have the following result.

\[
\text{Theorem 2} \quad \text{Let} \quad z_N, a, \quad \text{be the characteristic impedance at the Nth stage of construction of the SG ladder with} \quad \epsilon > 0 \quad \text{added to the impedances. Then,} \quad \lim_{N \to \infty} \lim_{\epsilon \to 0} z_{N,a} = z.
\]

The Hanoi circuit has two characteristic impedances, \( z_1 \) between \( p_0 \) and the central point of the Y-shape and \( z_2 \) between \( p_1 \) and the central point. An easily stated special case of our results on this circuit is as follows.

\[
\text{Theorem 3} \quad \text{If we require that impedances are scaled by a factor of} \quad \frac{1}{2} \quad \text{from level to level then the Hanoi circuit has characteristic impedances} \quad z_1 = \frac{1}{\sqrt{2}} \quad \text{and} \quad z_2 = 2\sqrt{L/C}.
\]

\[
\text{Harmonic Functions and Extension Matrices} \quad \text{References}
\]

The corresponding matrix for the Hanoi graph is unwieldy, but in the special case from Theorem 3 the eigenvalues \( \lambda_i \) and eigenvectors \( c_j \) are:

\[
\lambda_0 = 1, \quad c_0 = 1 \quad ; \quad \lambda_1 = \frac{(z_1 + z_2)(z_1 + 2z_2)}{3z_1^2(2z_1 + z_2) + z_2(z_1 + 2z_2 + 2z_2)} , \quad c_1 = \begin{cases} 0 & \text{if} \quad \lambda_2 = z_1 + z_2 + 2z_2, \\ -1 & \text{if} \quad \lambda_2 = \frac{z_1 + z_2 + 2z_2}{z_1 + 2z_2}, \end{cases}
\]

Any function on the vertices of the Hanoi is a linear combination of these three harmonic functions, and the intermediate vertices in each level have values that are weighted averages of the vertices already determined.

\[
\text{In any AC circuit, complex power is the product of voltage and current. If the potentials at all points in the circuit are stored in a vector} \quad Q, \quad \text{the complex power dissipation is defined by} \quad P = (EQ)^T (CEQ) = Q^T E^T C EQ
\]

where \( E \) is a vertex-edge transference matrix sending the potentials at vertices to the potential differences across edges, and \( C \) is a diagonal conductance matrix sending edge voltage to edge current according to Ohm’s law.

\[
\text{Theorem 5} \quad \text{The operator} \quad D = E^T C E \quad \text{that sends potential to total power dissipation on each circuit is invariant under network reduction upon taking the Schur complement.}
\]

For the SG circuit with characteristic impedance \( z \), the simplest power dissipation operator is

\[
D = \frac{1}{2} \begin{vmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{vmatrix}
\]

For the Hanoi circuit with characteristic impedances \( z_1 \) and \( z_2 \), the simplest power dissipation operator is

\[
D = \frac{1}{2z_1 + z_2} \begin{vmatrix} 2 & -1 & -1 \\ -1 & \frac{z_1 + z_2}{z_2} & -\frac{z_1}{z_2} \\ -1 & \frac{z_1 + z_2}{z_2} & \frac{z_1}{z_2} \end{vmatrix}
\]

Real power can be similarly obtained from the real part of potential energy with the operator \( \text{Re}(D) \), which is also constant from level to level, confirming that energy is conserved through the infinite substitution process.

We can also use our harmonic extension matrices to discover where in the circuit power is dissipated. This computation yields a complex analogue of the Kusuoka measure for the fractal ladder circuits.

\[
\text{REFERENCES} \quad \text{Acknowledgements}
\]

This work was completed in conjunction with our advisor Dr. Luke Rogers, during the 2013 University of Connecticut Research Experience for Undergraduates, funded by the NSF grant number 1262929 (DMS). Other contributors to this project are E. Akkermans, U. Andrews, E. Amiano, A. Brzoska, J. Chen, A. Coffey, G. Dunne, M. Dworken, L. Fisher, M. Harsalka, S. Law, and A. Teplyaev.