Investigating the Interplay of Argumentation and Mathematics in Classroom Tasks

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Motivation

Argumentation has been highlighted as an important skill for mathematical learning, specifically with the new Common Core State Standards (CCSS). The third standard for Mathematical Practice (MP3) in the CCSS stresses argumentation as an expertise all students should develop (CCSS, 2015). The NCTM Process Standards have been emphasizing its importance as a crucial element of learning for helping students clarify mathematical concepts and communicate them with others (NCTM, 2000). Argumentation is also recognized internationally as an important skill as reflected in examinations, such as TIMSS and PISA, that require students to construct arguments.

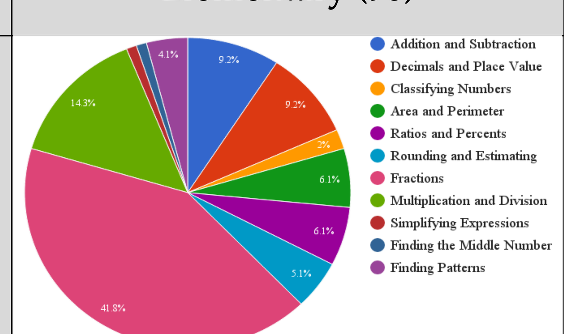
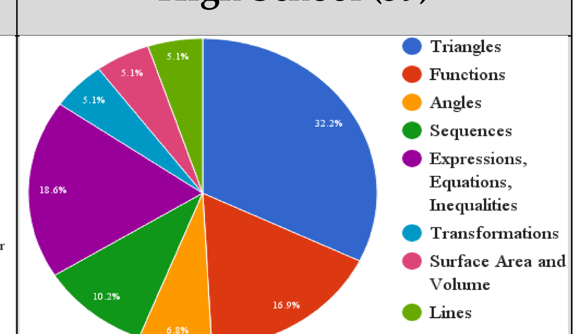
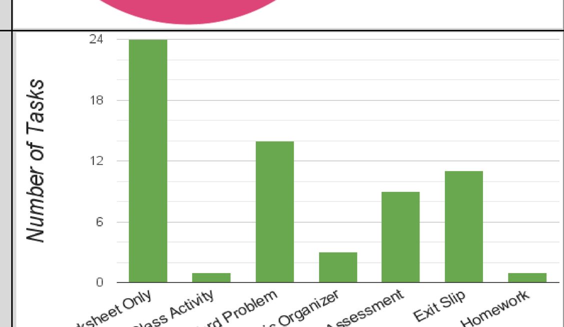
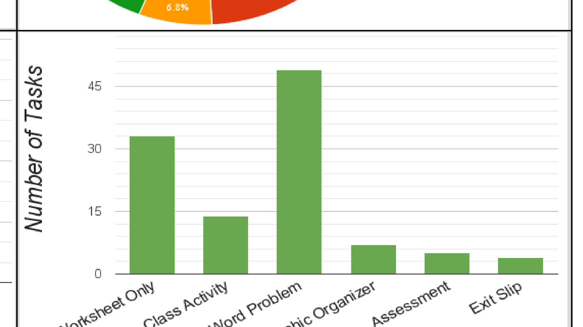
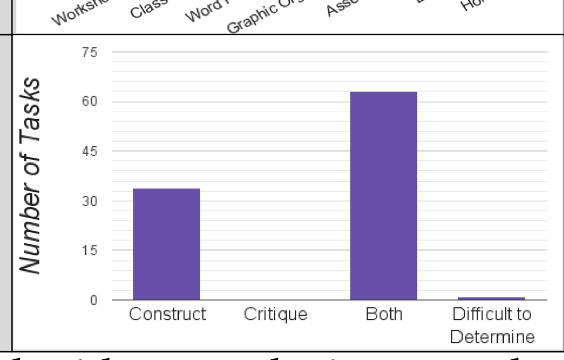
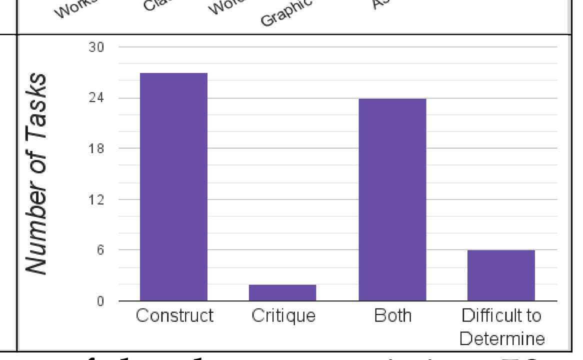
Purpose and Research Questions

Motivated by the current emphasis on argumentation, the purpose of our study is to discover the relationship between argumentation and mathematical concepts in tasks used by teachers in the classroom. Our research questions are

1. How is the cognitive demand level of mathematical tasks affected by argumentation?
2. How does argumentation detract from the mathematical content in the tasks?
3. How does argumentation contribute to the mathematical content in the tasks?

Methods

Our data set consists of 157 tasks from 40 teachers in a PD program. The table below shows sample results from a preliminary analysis of the tasks according to Grade Level, CCSS Domain, Math Topic, Type of Task, and Argument Type.

	Elementary (98)	High School (59)
Math Topic		
Type of Task		
Type of Argument		

This was followed with an analysis on a subset of the data containing 53 tasks (38 elementary and 15 high school) that were implemented in the classroom. Our analysis to address our research questions consisted of the following:

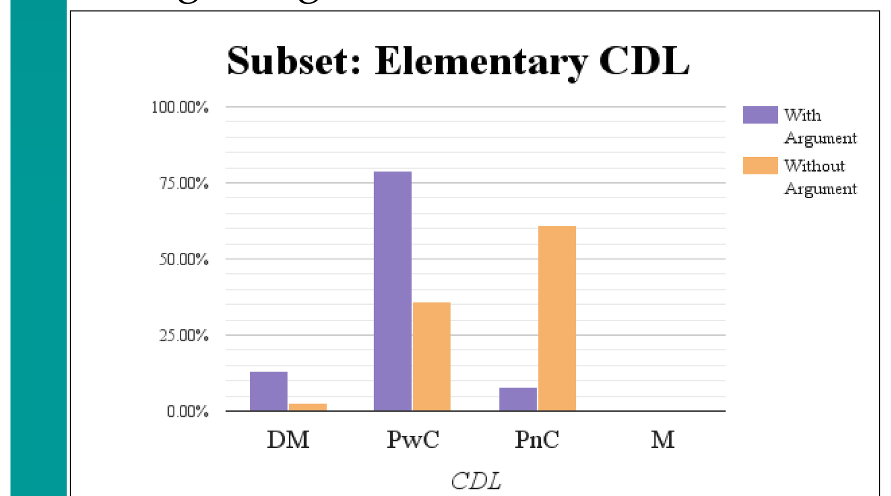
(i) We used the Task Analysis Guide (Stein, et al., 2000) to code the cognitive demand level (CDL) of the tasks' original form and then reevaluated the CDL without the argumentation component.

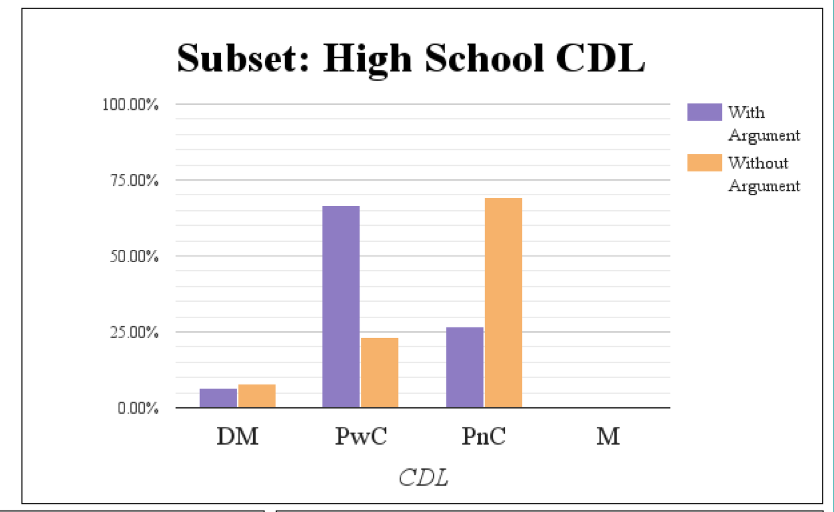
(ii) In analyzing the nature of the argument, we defined procedural tasks as those where the argumentation prompted students to explain their procedural steps and conceptual tasks as those that prompted students to consider an underlying topic in depth. Tasks were labeled as Both if the argumentation prompt was too open to categorize.

(iii) We analyzed whether the presence of the argumentation could potentially hinder the mathematical thinking or content in the tasks, as well as how the argumentation contributes to students' mathematical thinking and its impact on the resulting task when it is removed to identify patterns and themes.

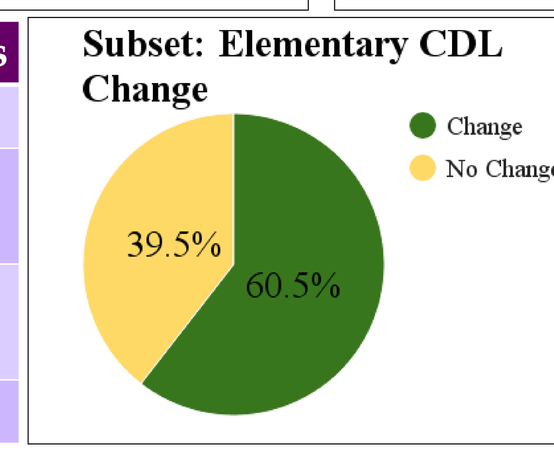
Findings

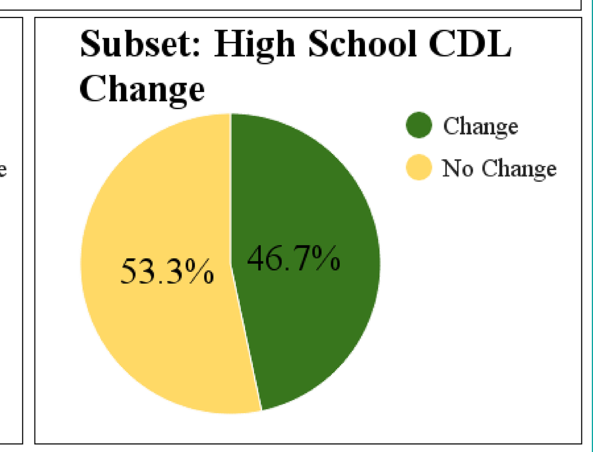
Research Question 1: Without the argumentation, tasks either remained at the same CDL or decreased. Some tasks' CDL could not be coded without argumentation, which led to a slight increase in percentage of high school tasks coded as DM. A higher percentage of elementary tasks changed CDL when the argumentation was removed in comparison to high school tasks. Note that there were less high school tasks in the data set, and hence less variety. See CDL change diagrams below.





Cognitive Demand Levels	
DM	Doing Mathematics
PwC	Procedures with Connections
PnC	Procedures without Connections
M	Memorization





Research Question 2: We found only two tasks from the subset, one high school and one elementary, where the argumentation detracts from the mathematical content. The prompts in these tasks involved critiquing a hypothetical student's answer. Since students were not directed to derive the information for themselves, *the argumentation allowed for less autonomy on the part of the students' mathematical thinking*.

Research Question 3: We identified five themes each from the elementary tasks and from the high school tasks that address the benefits argumentation posed in the problems. All themes are explained and illustrated in the following tables. Three themes overlapped in both elementary and high school tasks, as shown in the first table.

Theme	Description	Example
Common Themes		
Higher Order Thinking	Students are prompted to think at a higher level and make their conceptual knowledge of a specific topic more explicit. Students are led toward deepening their understanding of the concept rather than focusing on procedures.	Students critique arguments about vertical angles and reach higher-order thinking by critiquing a trial and error approach against an approach using supplementary angles, as well as by constructing their own argument. (HS Task)
Coherent Organization	Students are led to organize their thinking in a structured, coherent manner and are prompted to make connections within their own thinking.	Students use manipulatives to solve an addition problem and put their thoughts into words as to what they did to solve the problem. (Elem Task)
Justifying Procedures	Students use a series of calculations or procedures to solve a problem and are prompted to think about the reasoning that validates the procedures.	Students analyze a method used to solve a linear equation and explain the reasoning behind the procedural steps, helping students think about different properties for solving equations. (HS Task)

Theme	Description	Example
Elementary		
Tying Concepts Together	Students are led, with multiple questions, toward the construction of an argument which help them tie together the different concepts within the task.	Through multiple questions, students subtract and calculate equivalent fractions. Students justify their answer to a final question by drawing upon previous steps.
Analyzing Misconceptions	Students are led to recognize misconceptions that occur while applying algorithms or mathematical generalities and analyze them to better understand the mathematics.	Students must construct an argument to explain the correct reasoning behind multiplying fractions, which has them think more deeply about the misconception that multiplying fractions results in a larger fraction.
High School		
Connections Between Representations	Students are prompted to make connections between multiple representations, answers, and approaches to finding an answer.	Given an equation and three possible representations of the inverse (equation, table, and graph), students argue whether they represent the inverse of the function.
Connections to Prior Knowledge	Students are asked to recall previous knowledge to help defend their reasoning and make connections to new concepts within the problem.	Students identify increasing and decreasing functions from a table and must recall what they have previously learned about functions to defend their reasoning.

Discussion

- Argumentation contributes to higher-level thinking, as seen in the decrease in CDL of tasks without the argumentation component.
- All CDLs are represented (except M), indicating that tasks involving argumentation can be used for many different purposes in the classroom (Stein, et al., 2000).
- Disadvantages from the argumentation component can also be seen as a different way of addressing the content. When no disadvantages were found, the reasons reflected the advantages of argumentation.
- Themes found are supported in existing literature addressing argumentation in mathematics, including coherent organization, justifying procedures, analyzing misconceptions, and connections to prior knowledge (Boero, 1999; Cross, 2009; NCTM, 2000; Yackel, 1996).
- Possibilities for future research include opportunities for tasks to move further in addressing MP3 and reaching higher CDLs. Similar studies need to be conducted in different contexts. In addition, studies can also focus on implementation of tasks in the classroom.

References

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