Motivation

Argumentation has been highlighted as an important skill for mathematical learning, specifically with the new Common Core State Standards (CCSS). The third standard for Mathematical Practice (MP3) in the CCSS stresses argumentation as an expertise all students should develop (CCSS, 2015). The NCTM Process Standards have been emphasizing its importance as a crucial element of learning for helping students clarify mathematical concepts and communicate them with others (NCTM, 2000). Argumentation is also recognized internationally as an important skill as reflected in examinations, such as TIMSS and PISA, that require students to construct arguments.

Purpose and Research Questions

Motivated by the current emphasis on argumentation, the purpose of our study is to discover the relationship between argumentation and mathematical concepts in tasks used by teachers in the classroom. Our research questions are:

1. How is the cognitive demand level of mathematical tasks affected by argumentation?
2. How does argumentation detract from the mathematical content in the tasks?
3. How does argumentation contribute to the mathematical content in the tasks?

Methods

Our data set consists of 137 tasks from 40 teachers in a PD program. The table below shows sample results from a preliminary analysis of the tasks according to Grade Level, CCSS Domain, Math Topic, Type of Task, and Argument Type.

## Table

<table>
<thead>
<tr>
<th>Math Topic</th>
<th>Type of Task</th>
<th>Grade Level</th>
<th>Argument Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elementary (8th)</td>
<td>Example</td>
<td>High School (9th)</td>
<td>Explanation</td>
</tr>
</tbody>
</table>

This was followed with an analysis on a subset of the data containing 53 tasks (38 elementary and 15 high school) that were implemented in the classroom. Our analysis to address our research questions consisted of the following:

- We used the Task Analysis Guide (Stein et al., 2000) to code the cognitive demand level (CDL) of the tasks’ original form and then reevaluated the CDL without the argumentation component.

## Findings

### Research Question 1: Without the argumentation, tasks either remained at the same CDL or decreased. Some tasks’ CDL could not be coded without argumentation, which led to a slight increase in percentage of high school tasks coded as DM. A higher percentage of elementary tasks changed CDL when the argumentation was removed in comparison to high school tasks. Note that there were less high school tasks in the data set, and hence less variety. See CDL change diagrams below.

### Research Question 2: We found only two tasks from the subset, one high school and one elementary, where the argumentation detracts from the mathematical content. The prompts in these tasks involved critiquing a hypothetical student’s work. All CDLs are represented (except M), indicating that tasks involving argumentation can be used for many different purposes in the classroom (Stein, et al., 2000).

### Research Question 3: We identified five themes each from the elementary and high school tasks from the high school tasks that address the benefits argumentation posed in the problems. All themes are explained and illustrated in the following tables. Three themes overlapped in both elementary and high school tasks, as shown in the first table.

<table>
<thead>
<tr>
<th>Theme</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Higher Order Thinking</td>
<td>Students are prompted to think at a higher level and make their conceptual knowledge of a specific topic more explicit. Students are led toward deepening their understanding of the concept rather than focusing on procedures.</td>
<td></td>
</tr>
<tr>
<td>Coherent Organization</td>
<td>Students are led to organize their thinking in a structured, coherent manner and are prompted to make connections within their own thinking. Students use manipulatives to solve an addition problem and put their thoughts into words as to what they did to solve the problem. (Elem Task)</td>
<td></td>
</tr>
<tr>
<td>Justifying Procedures</td>
<td>Students use a series of calculations or procedures to solve a problem and are prompted to think about the reasoning that validates the procedures. Students analyze a method used to solve a linear equation and explain the reasoning behind the procedural steps, helping students think about different properties for solving equations. (Elem Task)</td>
<td></td>
</tr>
</tbody>
</table>

Discussion

- Argumentation contributes to higher-level thinking, as seen in the decrease in CDL of tasks without the argumentation component. However, some disadvantages were found.
- Disadvantages from the argumentation component can also be seen as a different way of addressing the content. When no disadvantages were found, the reasons reflected the advantages of argumentation.
- Themes found are supported in existing literature addressing argumentation in mathematics, including coherence, organizing, justifying procedures, analyzing misconceptions, and connections to prior knowledge (e.g., Cross, 2009; Stein, 2000; Yackel, 1995).
- Possibilities for future research include opportunities for tasks to move further in addressing MP3 and reaching higher CDLs. Similar studies need to be conducted in different contexts. In addition, studies can also focus on implementation of tasks in the classroom.

References


We gratefully acknowledge the support of NSF grant DMS 1262929; the Department of Mathematics at the University of Connecticut, Dr. Luke Haines for supervising this program, the teachers from the BPCME grant, and Kyle Evans, Megan Brunner, and Christopher Bennett.