# Spectrum of the Magnetic Laplacian

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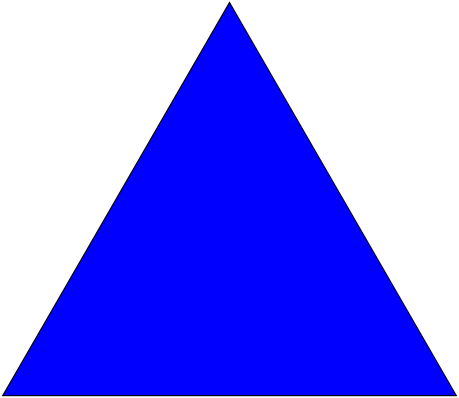
# Sierpinski Gasket



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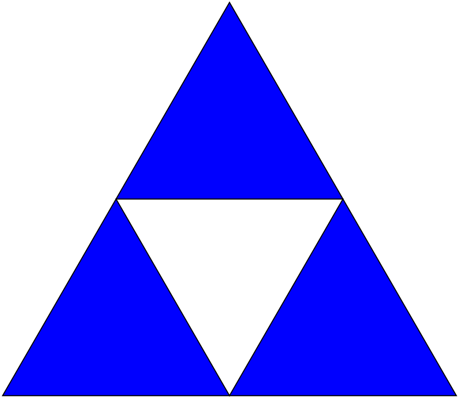

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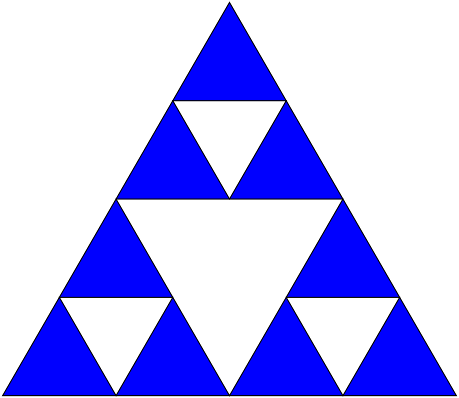
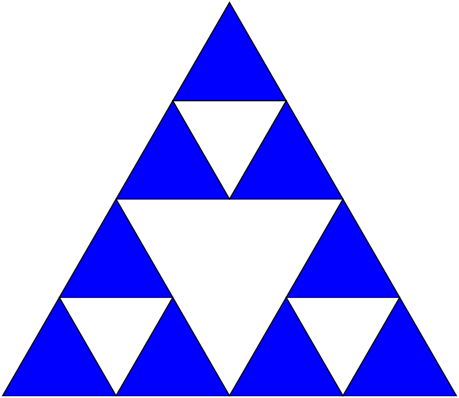
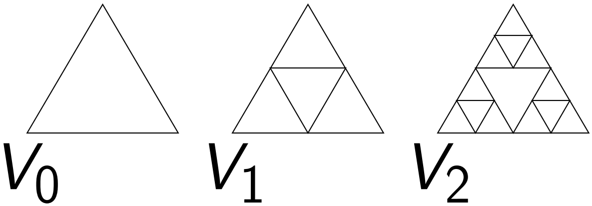


# Sierpinski Gasket






And so on...

# Graph Laplacian

The Laplacian, $\Delta$, holds information about the nature of a graph.

$$\Delta_n f(x) = \sum_{y \sim x} (f(x) - f(y))$$

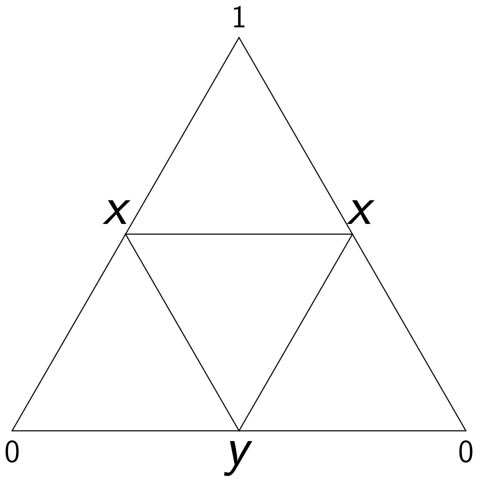
$$\Delta = \lim_{m\to\infty} 5^m \Delta_m$$

# Harmonic Functions

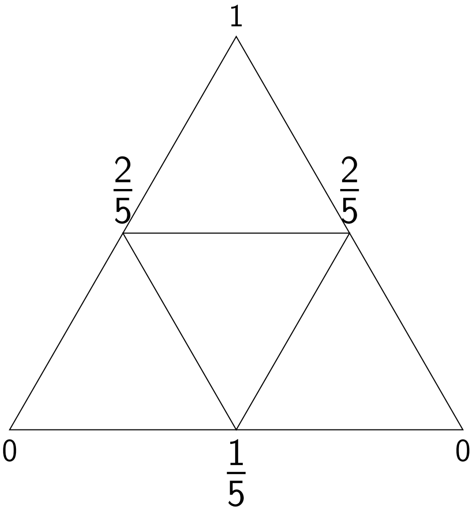
We say $h(x)$ is a harmonic function on $SG$ if $\Delta h(x) = 0, \forall x \in SG$

# Boundary Conditions

Harmonic function $h$ with fixed boundary conditions.

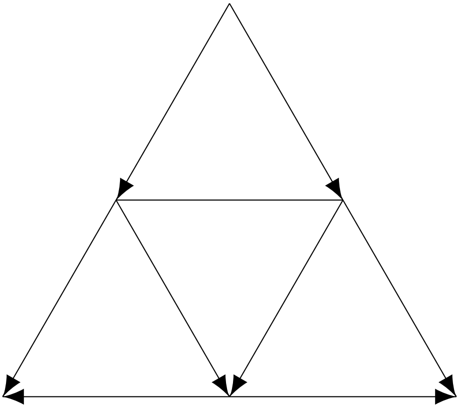


# Boundary Conditions

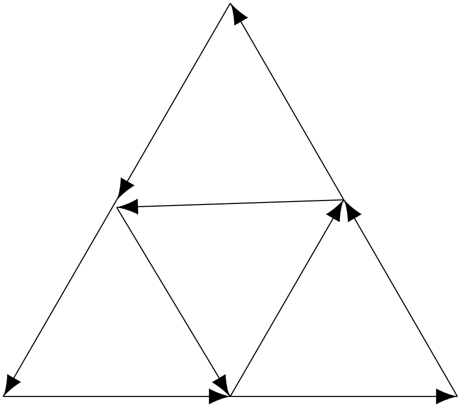


# Harmonic Form

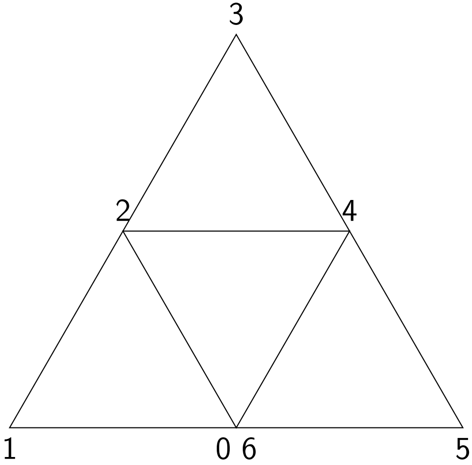
A 1−form is the analogue of a vector field, and has values on [directed] edges. The form shown is the exterior derivative of the harmonic function on the previous slide. Its value on the edge from $x$ to $y$ is $h(x) - h(y)$.



An example of a 1−form that is not the exterior derivative of a harmonic function.



# Harmonic Form

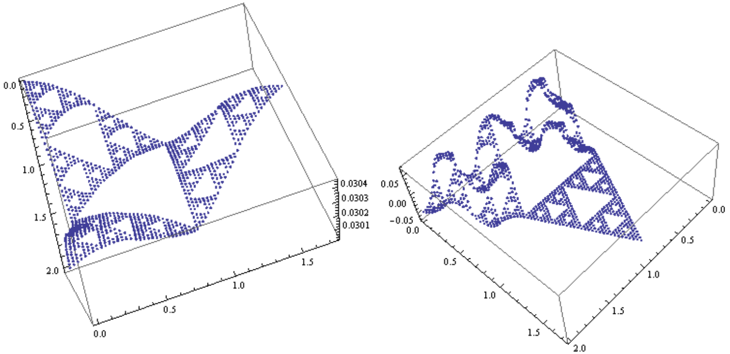


$$M_n^{\alpha} f(x) = \sum_{y \sim x} f(x) - e^{i\alpha A} f(y)$$

$$A(x,y) = h(x) - h(y)$$

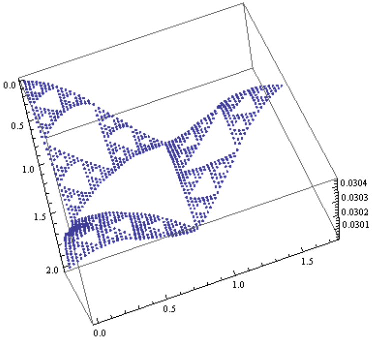
$$M^{\alpha A} = \lim_{m \to \infty} 5^m M_m^{\alpha A}$$

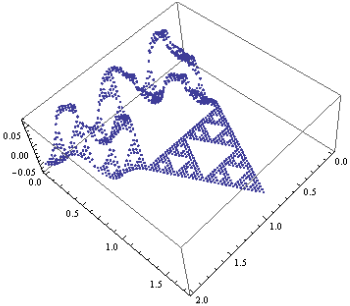
# Spectrum of the Magnetic Operator




Research goals:

# Spectrum of the Magnetic Operator

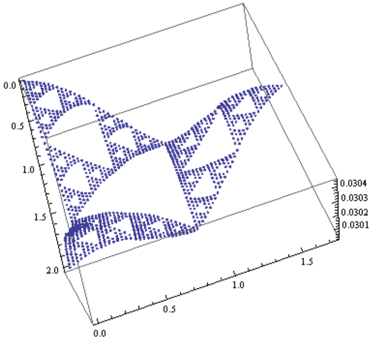


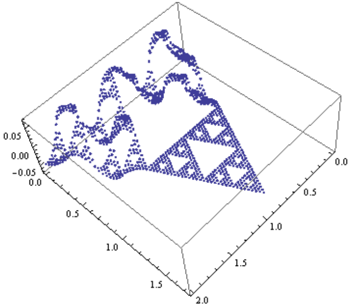




Research goals:

- Eigenfunctions

# Spectrum of the Magnetic Operator



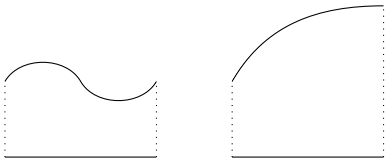

Research goals:

- Eigenfunctions
- Eigenvalues

# Gluing

We are able to find eigenfunctions of $M_m^{\alpha A}$. These solutions are found on the cut gasket, however. To bring these solutions back to $SG$ we need to do a gluing similar to a gluing in a Calculus 1 course.

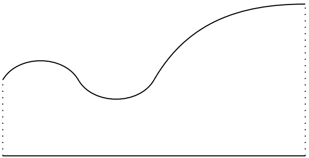
# Gluing from Calc 1



$f_1 : I_1 \to \mathbb{R}$ $\quad$ $f_2 : I_2 \to \mathbb{R}$

Say we want to join two functions $f_1$ and $f_2$ at point $p$ where $p = I_1 \cap I_2$.

# Gluing from Calc 1

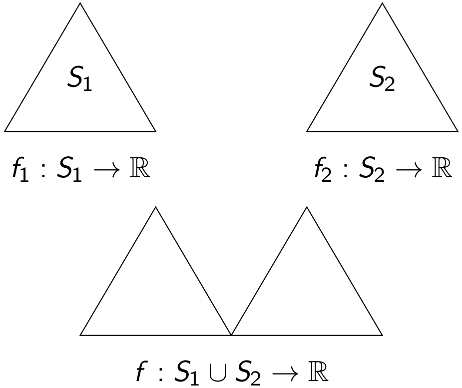


$$f : I_1 \cup I_2 \to \mathbb{R}$$

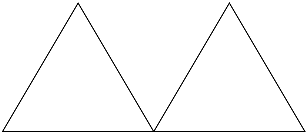
- Continuous: $f_1(p) = f_2(p)$
- Differentiable: $f_1(p) = f_2(p)$ and $f_1'(p) = f_2'(p)$
- Twice Differentiable: $f_1(p) = f_2(p)$, $f_1'(p) = f_2'(p)$, and $f_1''(p) = f_2''(p)$
- etc.

# Gluing Functions on Subcells of *SG*

Suppose we have functions on subcells of $SG$ that are to be joined at point $p = S_1 \cap S_2$.



# Gluing Functions on Subcells of $SG$



$$f : S_1 \cup S_2 \to \mathbb{R}$$

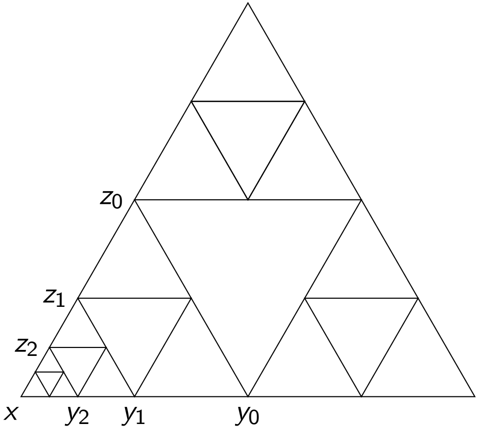
Our joined function $f$ will have a continuous Laplacian if

1. $f_1(p) = f_2(p)$
2. $\partial_n f_1(p) + \partial_n f_2(p) = 0$
3. $\Delta f_1(p) = \Delta f_2(p)$

Note: If $f_1$ and $f_2$ are eigenfunctions of $\Delta$ with the same eigenvalue then $1 \implies 3$. We then only need to check conditions 1 and 2.

# Normal Derivative

$$\partial_n f(x) := \lim_{m\to\infty} \left(\frac{5}{3}\right)^m [2f(x) - f(y_i) - f(z_i)]$$



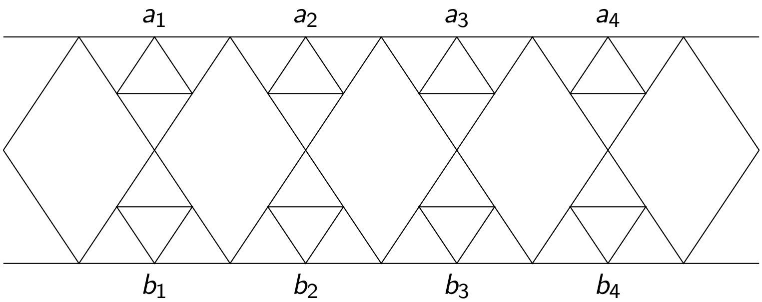
Suppose we are on a subcell of $SG$ and the magnetic field is $dA$ for a harmonic function $A$. Then our operator becomes

$$M_m^{\alpha A} = e^{i\alpha A}\Delta e^{-i\alpha A}.$$

It follows that if $f$ is an eigenfunction of $\Delta$ on this subcell with eigenvalue $\lambda$ then $g = e^{-iA}f$ is an eigenfunction of $M_m^{\alpha A}$ with the same eigenvalue $\lambda$.

We obtain eigenfunctions of $M_m^{\alpha A}$ on all of $SG$ by gluing the eigenfunctions of subcells.

# Infinite Ladder Sierpinksi Fracafold



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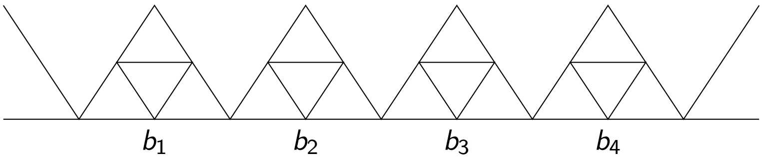

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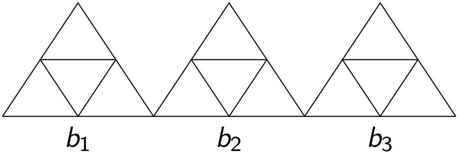

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