An Inquiry-Based Approach to Teaching Parameterization

Nicole DeMatteo, Jonathan Dollar, Gabriel Feinberg, and Fabiana Cardetti
Mathematics Education REU Project
University of Connecticut
Summer 2013

BACKGROUND

In the summer of 2012 the Math Education REU team at the University of Connecticut found that students struggle with certain topics within multivariable calculus. Of the topics identified, parameterization of curves in space appeared. Encouraged by the research on inquiry-based learning (IBL) showing positive results in student performance and attitudes (Laurson, 2011), our research team is convinced that an IBL activity could better assist students to understand parameterization. We have found in our literature review that IBL has been shown to be an effective teaching approach to spark students' curiosity and to help them develop a deeper understanding of the subject (Brickman, Gormally, Armstrong, & Hallar, 2009; Li, Moorman, & Dyjur, 2010; Zion & Mendelovici, 2012). Moreover, existing IBL research has focused on social work degree programs (Plowright & Watkins, 2004) and high school English and science courses (Brown, 2004; Chabalengula & Mumba, 2012; Zion et al., 2012). However, few studies have been conducted on implementing IBL in the undergraduate mathematics classroom. Furthermore, we found no research studies related to the use of IBL in multivariable calculus courses. In our study we address this gap, specifically for parameterization of curves, by creating an IBL activity along with supporting instructional guidance for its effective classroom implementation.

METHODS

IBL Definition

The literature does not provide a single, cohesive definition of IBL, thus our research team adopted a definition of IBL, in agreement with Yoshinobu and Jones (2012), as a general framework that encourages students to collaborate and become actively engaged in their learning experience.

Study Design

To address the purpose of this study, our team consulted with experienced instructors of multivariable calculus. Furthermore, we conducted a focused literature review and studied popular multivariable calculus textbooks regarding parameterization. From this, we developed a module grounded in theory, informed by the knowledge of our research team, and supported by the views of experienced instructors.

The Module

The Module encompasses:

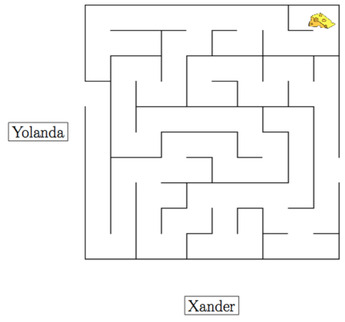
- A student handout composed of questions invoking upon IBL principles.
- An instructor guide to assist in inspiring mathematical student discovery, promoting student participation in group work, and facilitating class discussion.

RESULTS

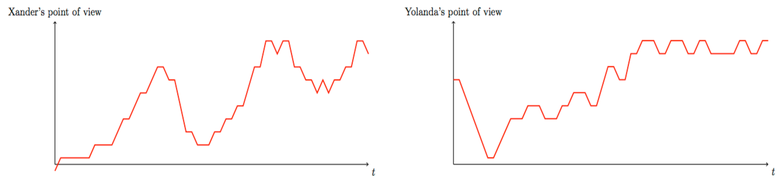
Our module advises the instructor to carefully listen to small groups working on the handout and to raise motivating questions at strategic moments.

Xander and Yolanda are two scientists observing a mouse in the following maze.

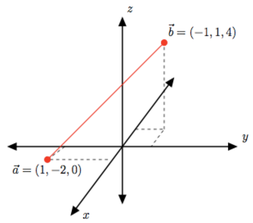
(a) Does the mouse get the cheese? If so, mark on the graphs when.
(b) Does the mouse escape from the maze? If so, mark on the graphs when.
(c) Trace the mouse's adventure through the maze.



The following are graphs of their perspectives of the mouse's position, t seconds after the start.



- In this question our module aims to introduce the concept of parameterization to the class in an interesting and natural way. As students ourselves we know this addresses the question of "where does this come from?"
- Students are used to expressing a curve as y=f(x), but in this question they are encouraged to think about expressing both coordinates of the points along a curve as functions of an additional variable. Multivariable calculus instructors identify this difference as a key concept within parameterization.



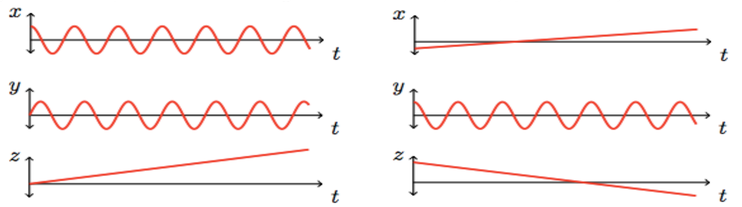
Excerpt from Instructor Guide

- Once all of the groups complete this question, ask one student to present the group's parameterization to the class.
- If you hear that a group parameterized the line segment over a different interval length, ask a student representative from the group to present to the class. Otherwise, present an alternative parameterization to the class yourself. Ask the class if both parameterizations work. Give them time to think and respond.
- Tell the class to think of the segment as the path a particle takes traveling through space. Ask at which speed it travels in each parameterization. Allow time for the students to think and discover the different rates.
- Ask the students if they can parameterize the line segment such that the particle is traveling at a non-constant speed. Give them time to try this task.
- Ask them to parameterize the segment if the particle travels the other direction.

- This component of the module relies heavily on instructor-guided discussion in order to extract as much as possible from the question. The literature shows that skillful guidance from the instructor is an integral component of IBL (Hmelo-Silver, Duncan, & Chinn, 2007; Speer & Wagner, 2009).
- Multivariable calculus instructors agree that it is important to emphasize parameterizing the same curve in multiple ways – over varied interval lengths, at constant/non-constant speed, or in different directions.

RESULTS

In this question, students are asked to match the graphs of a set of parametric equations to a general description of the resulting curve such as coil, wave, quadrilateral, etc.



- The literature reveals that generating multiple graphical representations in order for students to explain which option is correct is useful in directing mathematically relevant classroom discussions (Marrongelle & Rasmussen, 2008).
- This question aims at developing students' ability to visualize curves in 3D. This is an aspect of multivariable calculus that instructors agree to be challenging for students and crucial later on in the curriculum.
- While this question may not reflect the traditional use of parameterization within the calculus classroom, it emphasizes an important aspect of the IBL approach, advocating for deep conceptual understanding in addition to procedural practice (Yoshinobu & Jones, 2012).

FUTURE RESEARCH

Further research is necessary to determine the short- and long-term effectiveness of the implementation of the module with regards to students' learning of parameterization and instructor and student perceptions of learning with this module. Such research might also inform optimal placement of the module within the multivariable calculus curriculum; possible extensions such as other questions, applications, or topics; and supplemental computer lab activities. Future explorations could also focus on the impact of environmental components, such as class demographics, university environment, student-teacher interactions, and type of class.

REFERENCES

Brickman, P., Gormally, C., Armstrong, N., & Hallar, B. (2009). Effects of inquiry-based learning on students' science literacy skills and confidence.

Brown, H. (2004). Walking into the unknown: Inquiry-based learning transforms the english classroom. *The English Journal, 94*(2, Subversive English), 43-48.

Chabalengula, V. M., & Mumba, F. (2012). Inquiry-based science education: A scenario on zambia's high school science curriculum. *Science Education International, 23*(4), 307-327.

Hmelo-Silver, C., Duncan, R. G., & Chinn, C. A. (2007). Scaffolding and achievement in problem-based and inquiry learning: A response to kirschner, sweller, and clark (2006). *Educational Psychologist, 42*(2), 99-107.

Laursen, S., Hassi, M., Kogan, M., Hunter, A., & Weston, T. (2011). *Evaluation of the IBL mathematics project: student and instructor outcomes of inquiry-based learning in college mathematics.* Online report. Retrieved from http://www.colorado.edu/eer/research/documents/IBLmathReportALL_050211.pdf

Li, Q., Moorman, L., & Dyjur, P. (2010). Inquiry-based learning and e-mentoring via videoconference: A study of mathematics and science learning of canadian rural students. *Educational Technology Research & Development, 58*(6), 729-753. doi: 10.1007/s11423-010-9156-3

Marrongelle, K., & Rasmussen, C. (2008). Meeting new teaching challenges: Teaching strategies that mediate between all lecture and all student discovery. In M. P. Carlson, & C. Rasumussen (Eds.), *Making the connection: Research and teaching in undergraduate mathematics education* (pp. 167-178). Washington DC: Mathematical Association of America.

Speer, N. M., & Wagner, J. F. (2009). Knowledge needed by a teacher to provide analytic scaffolding during undergraduate mathematics classroom discussions. *Journal for Research in Mathematics Education, 40*(5), 530-562.

Yoshinobu, S., & Jones, M. G. (2012). The coverage issue. *Primus, 22*(4), 303-316. doi:10.1080/10511970.2010.507622

Zion, M., & Mendelovici, R. (2012). Moving from structured to open inquiry: Challenges and limits. *Science Education International, 23*(4), 383-399.

A special thanks to NSF grant DMF 1262929, UConn, and all the expert instructors we consulted!